

Self-tuning Fuzzy Control Method Based on the Trajectory Performance of the Phase Plane¹

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Abstract: The phase plane is already an important method to design fuzzy control systems and analyze their stability. The concept of the real-time response trajectory characteristic vectors and angles between the real-time characteristic vectors on the phase plane are put forward in this paper according to the analysis of the response trajectory performance on the phase plane of a fuzzy control system. The method of rule self-tuning fuzzy control based on the response trajectory performance on phase plane is presented by analyzing the characteristics of angles between the real-time characteristic vectors. The simulation results show that the method is not only capable of increasing greatly the ability to identify and describe the plant in small error, reducing the overshoot, settle time greatly and improving the convergence speed of the fuzzy control system, but also possesses a simple arithmetic and does not require much more storage space and calculation time.

Key words: self-tuning fuzzy control; trajectory performance; phase plane.

1. INTRODUCTION

In designing fuzzy control system, phase plane has been widely applied in analyzing control system stability, designing fuzzy controller and increasing the control performance by tuning error, change-in-error and control output. Gu Shusheng and Ping Li firstly put forward phase plane method in analyzing stability of fuzzy control system in 1991^[1], then Yi and Chung realized a systematic procedure of design and analyzing stability of fuzzy controller based on phase plane^[2]. In order to study the analytic structure of fuzzy controller, papers^[3-8] have analyzed the analytic structures of fuzzy controllers and

pointed out that fuzzy controller is similar to a sliding mode variable structure controller theoretically. In order to increase the control performance of fuzzy control, Hwan-Rong Lin and Wen-June Wang combine the sliding mode control and fuzzy logic concepts to complete the phase plant trajectory task^[9]. According to the response trajectory performance on phase plane, Huang and Nelson advanced the method of initial rule tuning fuzzy control by analyzing dynamic response process of fuzzy controller in 1994^[10,11], it can reducing oscillation and limit-loop and improving the convergence speed of fuzzy control system greatly on rule-tuning. The simulation results show it is applicable for the fuzzy control of air-condition system. They developed self-tuning fuzzy controller on phase plane in 1999^[12]. They firstly divided phase plane into some region according to the response trajectory of the control system on phase plane and made the system response trajectory distribute in different region. Then gave the ideal response trajectory of the control system from start point to object point, and mark some reference point on phase plane. Finally adjusted the fuzzy control rule according to the distance between the practical response trajectory and different region reference point and made the fuzzy controller worked along the ideal response trajectory. The simulation and experimentation results show that the method has preferable control performance, but its design and tuning process are complicated, requires much more storage space and the calculating time. Aiming at those shortcomings in paper^[12], the concepts of the real-time response trajectory characteristic vector on phase plane of fuzzy control system is presented firstly in this paper, further the method of rule self-tuning fuzzy control based on the response trajectory performance on phase plane has been

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presented^[13]. The simulation results show that the method is not only capable of increasing the control performance, but also possesses a simply algorithm and does not require much more storage space and the calculating time.

2. RESPONSE TRAJECTORY PERFORMANCE ON PHASE PLANE AND REAL- TIME CHARACTERISTIC VECTORS OF FUZZY CONTROL SYSTEM

A block diagram of a fuzzy control system is shown in Figure 1, where T denotes sample period (s), r denotes the set point, Δu denotes the controller output, y denotes plant output. Equation (1) is the transfer function of plant, Figure 2 shows the membership functions, and Table 1 is the fuzzy control rule. The system response trajectory on phase plane is shown in Figure 3, where S_i ($i=1,2,\dots,n$) denote plant output at different sample instant, that is response trajectory; R denotes overshoot point; O denotes object point.

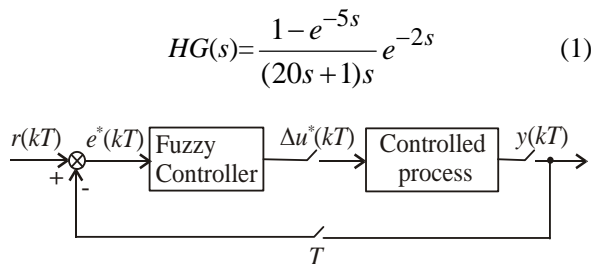


Fig.1 Fuzzy control system model

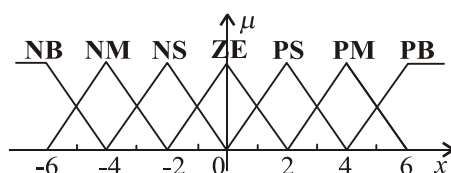


Fig.2 Triangle membership functions

As is shown in Figure 3, the system begins with $(e, ec) = (-3, 0)$, along the tracing trajectory S_i , successively passes through the second quadrant ($e < 0, ec > 0$), the first quadrant ($e > 0, ec > 0$), and the forth quadrant ($e < 0, ec < 0$), lastly converge at object point O(0,0) in a spiral line. After it approaches the set value ($e=0$) with a fast speed from the start point in second quadrant, the system gradually deviates from the object point in first quadrant, finally arrives at the overshoot point R because of larger inertia ($ec \approx 1.9$). Though it is able to approximate the set value in forth

quadrant quickly, the system has certain hysteretic near the object point. So the control action is very important when the system comes into the first quadrant from the second quadrant, and it is effective to control ahead or increase the control intension on reducing system overshoot and hysteretic characteristic.

Tab. 1 Fuzzy control rules

B_j		NB	NM	NS	ZE	PS	PM	PB
A_i	C_{ij}							
	NB	PB	PB	PM	PM	PS	PS	ZE
	NM	PB	PM	PM	PS	PS	ZE	NS
	NS	PM	PM	PS	PS	ZE	NS	NS
	ZE	PM	PS	PS	ZE	NS	NS	NM
	PS	PS	PS	ZE	NS	NS	NM	NM
	PM	PS	ZE	NS	NS	NM	NM	NB
	PB	ZE	NS	NS	NM	NM	NB	NB

Note: A_i , B_j and C_{ij} ($i, j=1,2,\dots,7$) are fuzzy subsets of error e , change-in-error ec and output Δu

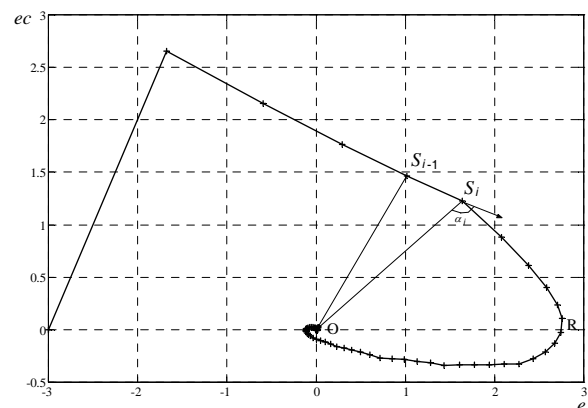


Fig. 3 Response trajectory on phase plane of some fuzzy control system

In Figure 3, Suppose that S_i denotes present system response, S_{i-1} denotes previous system response, $\overline{S_{i-1}S_i}$ denotes the response trajectory real-time tracing vector on phase plane of the control system. It is obvious that the start point, end point and direction of the vector $\overline{S_{i-1}S_i}$ change with time, $\overline{S_{i-1}S_i}$ expresses the position and movement direction, indicates the real-time response characteristic. Similarly suppose S_i is start point,

object point O is end point, and then vector $\overline{S_iO}$ is the real-time response trajectory target vector on phase plane of the control system, denoting the real-time position and direction of the control system relative to the above two vectors are called the real-time characteristic vectors on phase plane of the control system. The angle α_i that between $\overline{S_{i-1}S_i}$ and $\overline{S_iO}$ is called the real-time response trajectory characteristic vector angle on phase plane of the control system. α_i shows the present changing direction of the control system is deviating or approximating from the object point, and indicates the two regulation influences on the control system.

In Figure 3 α_i is angel between vectors $\overline{S_{i-1}S_i}$, $\overline{S_iO}$, the coordinate of point S_{i-1} , S_i and O are (e_{i-1}, ec_{i-1}) , (e_i, ec_i) and (e_0, ec_0) respectively, thus Real-time tracing vector would be

$$\overline{S_{i-1}S_i} = \{e_i - e_{i-1}, ec_i - ec_{i-1}\} \quad (2)$$

The real-time target vector would be

$$\overline{S_iO} = \{e_0 - e_i, ec_0 - ec_i\} \quad (3)$$

The angles between the real-time characteristic vectors would be

$$\alpha_i = \arccos\left(\frac{\overline{S_{i-1}S_i} \cdot \overline{S_iO}}{|\overline{S_{i-1}S_i}| |\overline{S_iO}|}\right) \quad (4)$$

α_i could be obtained from Equation (2)~(4).

3. CHARACTERISTIC ANALYSIS OF THE ANGLES BETWEEN THE REAL-TIME CHARACTERISTIC VECTORS OF FUZZY CONTROL SYSTEM

Figure 4 displays the response trajectory characteristic vectors on phase plane of fuzzy control system which shown in Figure 1. From Figure 4, it can be found that if $\alpha_i > 90^\circ$, S_i deviates from the object point O more and more, then the control intensity should be increased; if $\alpha_i < 90^\circ$, S_i deflects from the object point O more and more, then the control intensity should be maintained or

appropriately increased; if $\alpha_i = 90^\circ$, that is $\overline{S_{i-1}S_i} \perp \overline{S_iO}$, S_i circles around the object point O, then the control intensity should be increased.

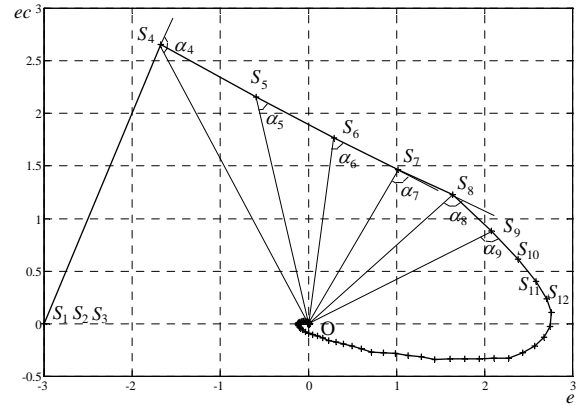


Fig. 4 Response trajectory performance vectors on phase plane of fuzzy control system

Let angle 90° be a benchmark, $\Delta\alpha_i$ denotes the error between the real-time characteristic vector and the reference, namely

$$\Delta\alpha_i = \alpha_i - 90^\circ \quad (5)$$

and α_{i-1} denotes previous angle between the real-time characteristic vectors, then $\Delta^2\alpha_i$ denotes the change-in-error of the angle between the real-time characteristic vectors in two adjacent time, namely

$$\Delta^2\alpha_i = \alpha_i - \alpha_{i-1} \quad (6)$$

See Table 2 for the angles between the real-time characteristic vectors $\Delta\alpha_i$ and their changes $\Delta^2\alpha_i$, where k is simulation numbers.

Notice that from Table 2, $\Delta\alpha_i$ specifies the state response of the control system deviate from or approximate to the object point, $\Delta^2\alpha_i$ specifies its tendency, namely

If $\Delta\alpha_i > 0$, system deviates from the object point, indicating that control intensity should be increased.

If $\Delta\alpha_i < 0$, system approximates to the object point, indicating that control intensity should be adjusted appropriately based on $\Delta\alpha_i$.

If $\Delta^2\alpha_i > 0$, system trends to deviate from the object point, indicating that control intensity should be increased.

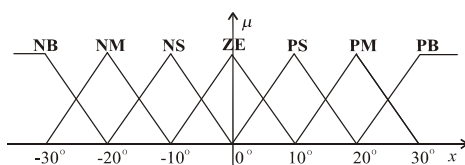
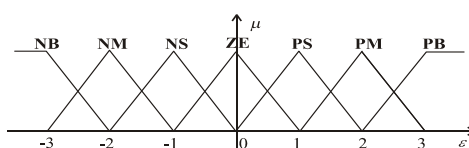
If $\Delta^2\alpha_i < 0$, system trends to approximate to the object point, indicating that control intensity should be adjusted appropriately based on $\Delta^2\alpha_i$.

Tab.2 The angles between the real-time characteristic vectors and their changes

k	$\Delta\alpha_i$	$\Delta^2\alpha_i$	System state
1	0°	0°	start point
2	0°	0°	start point
3	0°	0°	start point
4	31.2°	121.2°	deviate from object point
5	-40.3°	-71.5°	approximates to object point
6	-14.4°	25.8°	approximates to object point
7	12.4°	26.9°	deviate from object point
8	32.1°	19.7°	deviate from object point
9	28.8°	-3.3°	deviate from object point
10	34.1°	5.3°	deviate from object point
11	34.7°	0.64°	deviate from object point
12	30.8°	-3.9°	deviate from object point
13	20.8°	-9.9°	deviate from object point
14	-4.9°	-25.7°	approximates to object point
15	-29.1°	-24.2°	approximates to object point
16	-47.7°	-18.6°	approximates to object point

4. FUZZY METHOD TO MODIFY-CONSEQUENCE OF FUZZY CONTROL RULES

Consider Figure 5 which are membership functions of $\Delta\alpha_i$ and $\Delta^2\alpha_i$, defined its discourse domain is $[-30^\circ, 30^\circ]$, and its fuzzy subsets are {NB, NM, NS, ZE, PS, PM, PB}. Let the modified value for the consequence of fuzzy rule be ε , its discourse domain be $[-3, 3]$, its fuzzy subsets are {NB, NM, NS, ZE, PS, PM, PB}, its membership function be given by Fig 6.

**Fig. 5 Membership function of $\Delta\alpha_i$ and $\Delta^2\alpha_i$** **Fig. 6 Membership function of the modified variable for the consequence****Tab. 3 Adjusting table of the modified variable ε of the initial rule consequence**

$\Delta\alpha_i$	$\Delta^2\alpha_i$	NB	NM	NS	ZE	PS	PM	PB
NB	ZE	ZE	ZE	ZE	NS	NS	NM	
NM	ZE	ZE	ZE	NS	NS	NM	NM	
NS	ZE	ZE	NS	NS	NM	NM	NM	
ZE	ZE	NS	NS	NM	NM	NM	NB	
PS	NS	NS	NM	NM	NM	NB	NB	
PM	NS	NM	NM	NM	NB	NB	NB	
PB	NM	NM	NM	NB	NB	NB	NB	

According to the practical control requirements, the modification of the fuzzy grade for the consequence is only increasing, no decreasing. Then there is $\varepsilon \in [0, 3]$, and if $\varepsilon=0$, there is no modification. Table 3 represents the correction method of the initial rule consequence.

Thus based on $\Delta\alpha_i$, $\Delta^2\alpha_i$ and Table 3, we can get correction ε for the consequence at different instant, and accordingly modify initial reasoning result Δu . Consequently rule self-tuning process of fuzzy control is realized.

5. METHOD OF RULE SELF-TUNING FUZZY CONTROL BASED ON TRAJECTORY PERFORMANCE ON PHASE PLANE

As is shown in Figure 7 is rule self-tuning fuzzy controller which taking Functioning-fuzzy-subset Inference (FFSI)^[14] as the base, taking Equation (1) as the plant, taking Table 1 as the initial fuzzy control rule, taking Table 3 as the modified rule, adding performance evaluation, rule regulator, rule tuning and tuning rule on the basis of Figure 1. Tuning rule and initial rule module are given in Table 3 and Table 1 respectively. Consider Equation (7), which is index ITAE evaluating the performance of fuzzy control. During simulation, quantification factors of e and ec are 3 and 6 respectively, scale factor of Δu is 1/30. And Figure 8 and Figure 9 show the control results of rule self-tuning and FFSI.

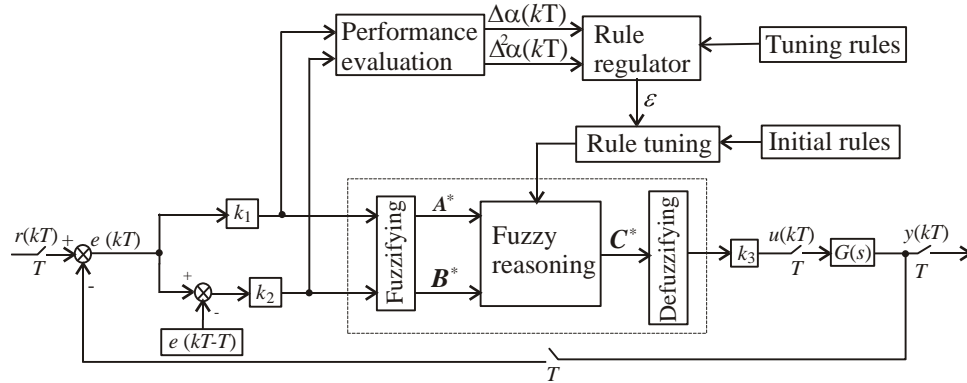


Fig. 7 Rule self-tuning fuzzy controller

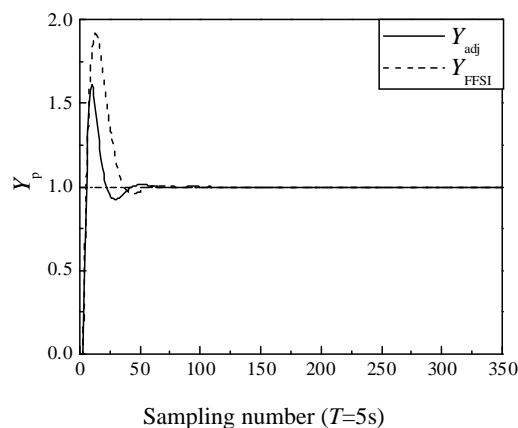


Fig. 8 Results of the fuzzy control in simulation

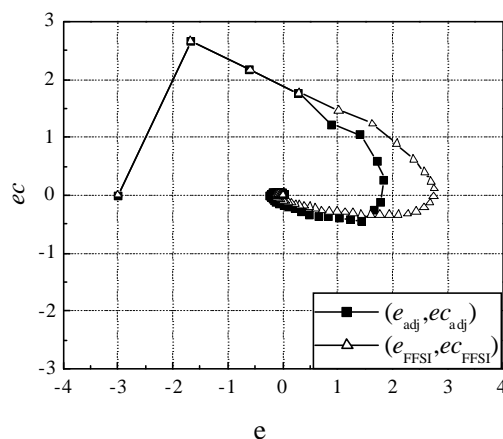


Fig 9 Phase plane trajectory of the fuzzy control

$$\text{ITAE} = \sum_{k=0}^n |e(kT)| \cdot kT \cdot T \quad (7)$$

Y_{adj} and Y_{FFSI} in Figure 8 show the fuzzy control results of rule self-tuning and FFSI respectively, where Y_p is the system unit step response. $(e_{\text{adj}}, ec_{\text{adj}})$ and $(e_{\text{FFSI}}, ec_{\text{FFSI}})$ in Figure 9 represent the system error and change-in-error under the fuzzy control of

rule self-tuning and FFSI respectively. The overshoot is reduced distinctly, about half of the initial, and the transient process is lessened after tuning rule, as is given in Figure 8. Consider Figure 9, almost at the sixth step, the system response trajectory obviously deflect to the object point, thus overshoot reduces and the convergence speed quickens.

Note that the index ITAE of FFSI is 1.450×10^3 , while that of self-tuning is 0.576×10^3 , is about 1/3 to the former.

6. CONCLUSIONS

The paper puts forward the concept of the response trajectory real-time characteristic vectors, and presents the method of rule self-tuning fuzzy control based on the response trajectory performance according to the analysis of the response trajectory on phase plane of fuzzy control system. We can get the following results by simulation.

(1) The response trajectory real-time characteristic vectors of control system can fully described the dynamic and static characteristics of fuzzy control system, especially by the error and change-in-error of angle between the real-time characteristic vectors, which greatly increases system identification ability under small deviation.

(2) The method of rule self-tuning fuzzy control based on the response trajectory performance can decrease IATE greatly, reduce the overshoot and settle time, improve the convergence speed of the control system.

(3) The method of rule self-tuning fuzzy control based on the response trajectory performance time, it is very easy to perform.

7. ACKNOWLEDGEMENTS

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